of the series solution are taken, finer interval along the boundary perimeter is divided into, and more floating point digits are carried on the computer. One of the biggest advantages in using this method is that very little work is involved in switching from one shape of plate to the next or in changing from one type of boundary conditions to the other, however mixed. The former involves changing the portion of the program that computes the locations of boundary points, whereas the latter involves only the changing of an input data card.

References

[1] Timoshenko, S. P., *Theory of Elasticity*, 2nd ed., McGraw-Hill, New York, 1951, p. 116.

[2] Morley, L. S. D., "Bending of a Simply Supported Rhombic Plate under Uniform Normal Loading," *Quarterly Journal of Mechanics and Applied Mathematics*, Vol. 15, Pt. 4, 1962.

[3] Chen, S. S. H. and Pickett, G., "Bending of Plates of Any Shape and with Any Variation in Boundary Conditions," *Journal of Applied Mechanics*, Vol. 34, No. 1, 1967, pp. 217–218.

[4] Chen, S. S. H. and Pickett, G., "Bending of Uniform Plates of Arbitrary Shapes and with Mixed Boundary Conditions," *Developments in Mechanics*, Vol. 4, 1968, pp. 411–427.

[5] Bailey, R. and Hicks, R., "Behaviour of Perforated Plates Under Plane Stress," *Journal of Mechanical Engineering Science*, Vol. 2, No. 2, 1960, pp. 143–161.

[6] Howland, R. C. J., "On the Stresses in the Neighbourhood of a Circular Hole in a Strip Under Tension," *Philosophical Transactions*, Vol. A229, 1930, pp. 49–85.

[7] Buchwald, V. T. and Davis, G. A. O., "Plane Elastostatic Boundary Value Problems of Doubly Connected Regions I," *Quarterly Journal of Mechanics and Applied Mathematics*, Vol. XVII, Pt. I, 1964, pp. 1–15.

Lift Reduction in Additive Solutions

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Introduction

It has been some time since the "rediscovery" (Hoyt and Fabula[1]) of Toms' phenomenon[2]: a large reduction of turbulent friction is obtained by adding extremely small quantities of high-molecular-weight additive to water. The potential practical application and the theoretical challenge have made this phenomenon one of the most important subjects for hydrodynamic research in recent years. Although an understanding of the physics governing this phenomenon is pursued actively at present,[3] the fact of large obtainable drag reduction is motivating engineers to look beyond the pipe-flow study and to undertake ship-model tests as well as prototype trials.

The motivation for using additives to reduce ship resistance is saving power or fuel from the economic point of view, and increasing speed from the performance point of view. However, the additive ejected around the ship will inevitably be introduced into the propeller section. The drag-reduction advantage can be greatly hampered if the additive produces adverse effects on the function of the propeller. This Note presents some observations, by-products of a drag reduction study,[4] on the possible thrust reduction of an impeller in additive solutions.

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Experimental Technique

The experiment was conducted in a circulating water channel with a closed test section 44 in. long, 15 in. wide, and $7\frac{1}{2}$ in. deep; see Fig. 1a. At the upstream end of the contraction is a honeycomb. Energy dissipators consisting of screens and grids are placed at the discharge end of the test section. The water is circulated in the channel by a 3-bladed, 14-in.-diam impeller, placed in a circular duct and driven by a 3-hp motor-generator set.

A part of the cover plate at the test section, 10 in. wide and 20 in. long, is cut from the rest with a clearance of $\frac{1}{100}$ in. along four sides. The drag of this part of the cover plate is measured with a strain gage. The velocity in the channel is determined by photographing a small, neutrally buoyant particle, released at the upstream end of the test section. The time exposure picture of the path of the particle, illuminated by a strobe light, is shown as a series of bright dots on film. By comparing the distance between dots and the time interval between flashes, the channel speed is determined.

The test fluid consists of aqueous solutions of polyethylene oxide (Polyox WSR-301) of various concentrations. Shear degradation of the testing fluid is inevitable and will be discussed in a later section.

Results

For the pure water case, the channel velocity at the test section, proportional to the velocity in the slipstream, is shown in Fig. 1b to vary linearly with the rotational speed of the impeller. The constant of proportionality can readily be determined. Part of the results of the drag-reduction study,[4] conducted simultaneously with the present experiment, are presented in Fig. 1c. These results provide some information about the present experimental conditions.

For tests with additive solutions, the ratio between the channel velocity and the rpm of the impeller was determined at various impeller speeds and compared with that of the pure water case (the proportionality constant determined previously). Several series of tests were conducted for each additive concentration. During each series, the channel speed

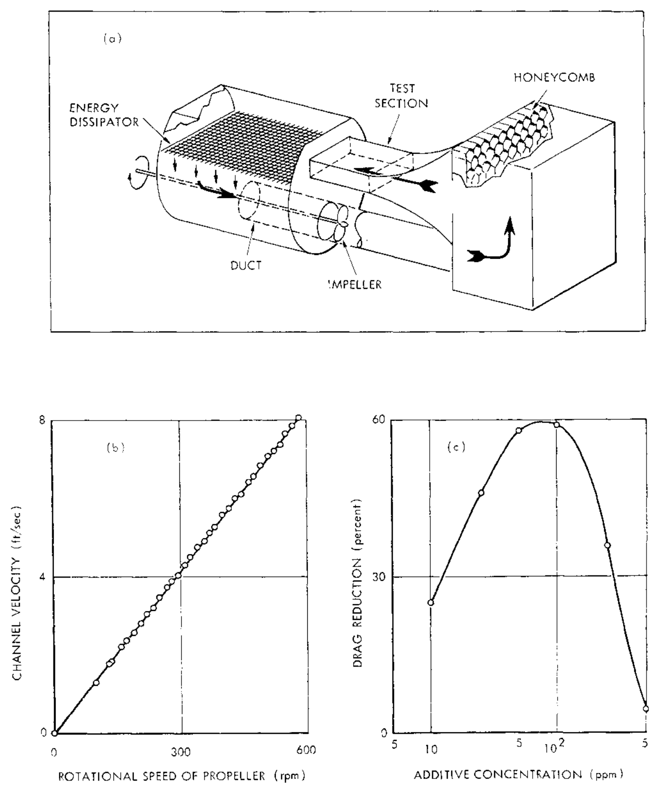

Fig. 1 General view and calibration of equipment.

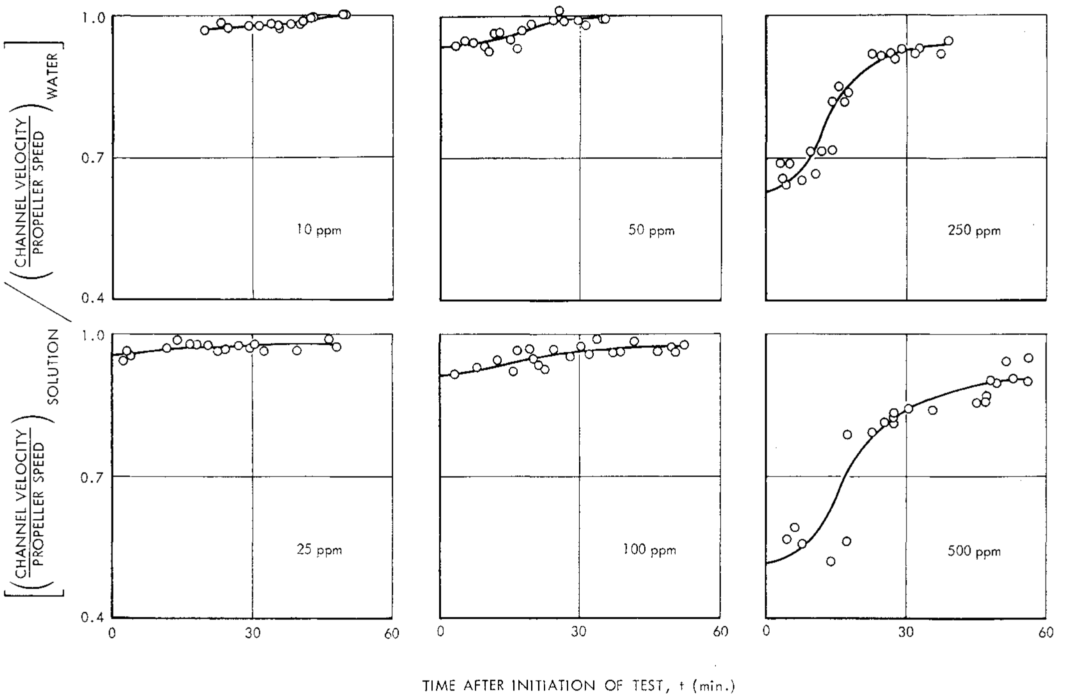

Fig. 2 Possible thrust reduction of impeller in additive solutions of various concentrations.

was varied monotonically with an alternate increase and decrease of speed from series to series.

The results, plotted vs the time after the initiation of pumping in Fig. 2, show that the ratio between the channel velocity and the rotational speed of the impeller is generally reduced in the presence of additives in water. Shear degradation of the additive solution is responsible for the temporal variation of this ratio shown in Fig. 2. The data display an s-shaped curve: the additive degrades rather slowly during the initial stage, a rapid degradation follows, and then the ratio approaches gradually and asymptotically that of the pure water case (a ratio of unity) as time passes. Extending these curves to time equal to zero, one obtains the initial ratios between the channel velocity and the impeller rotational speed at various additive concentrations before any shear degradation has taken place; see Fig. 3.

The reduction of the ratio between the channel velocity and the rotational speed of the impeller in additive solutions may be caused by the increase of channel resistance (includes the frictional resistance on the solid boundary and the form drag of obstructions), or by the flow separation from the impeller blades. However the reduction of the frictional resistance was observed directly in this experiment, whereas the reduction of the form drag, such as the drag of spheres in additive solutions, was reported by Lang and Patrick.[5] The latter study also indicates that the tendency of flow separation is reduced in additive solutions. Furthermore, as presented in Fig. 2, the trend of the data is to vary systematically with additive concentration and, moreover, the data also vary rather smoothly with time, approaching the pure water case asymptotically. If flow separation occurs one would expect an abrupt change of the experimental results from separating to nonseparating conditions. The foregoing considerations suggest that flow separation from the impeller blades most probably does not occur.

The viscosity of a dilute polyox-additive solution increases almost linearly with the additive concentration and is double the pure water value at 900 ppm.[6] However, the lift produced by the propeller blade should not be influenced by this change, which represents only a small change of Reynolds numbers. Moreover, any reduction of the Reynolds number usually increases, if any, the ratio between the slip-stream velocity and the rotational speed of the propeller.

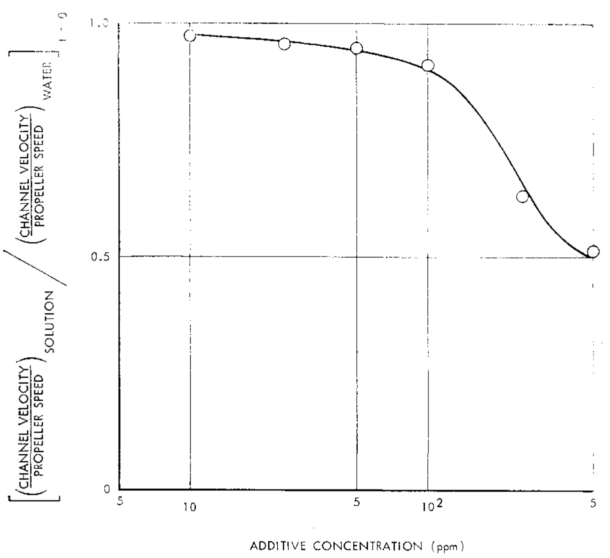

Fig. 3 Possible reduction of impeller thrust in additive solutions.

Concluding Remarks

The present results, a by-product obtained during a drag-reduction study, indicate the possibility of a reduction of impeller thrust in additive solutions. This observation may be important in view of the application of additive to reduce ship resistance, since the additive ejected around the ship may produce adverse effects on the propeller thrust. Studies to understand the present observations also may provide complementary insights to macromolecular theory of dispersive systems. An experiment aimed at studying this phenomena specifically, such as measuring the lift of a hydrofoil in additive solutions, is certainly worthwhile.

References

[1] Hoyt, J. W. and Fabula, A. G., "The Effects of Additives on Fluid Friction," *Proceedings of the 5th ONR Symposium on Naval Hydrodynamics*, Office of Naval Research, Bergen, 1964.

[2] Toms, B. A., "Some Observations on the Flow of Linear Polymer Solutions through Straight Tubes at Large Reynolds Numbers," *Proceedings of the 1st International Rheological Congress*, Holland, 1948.

[3] Wu, Jin, "Drag Reduction in External Flows of Additive Solutions," *Proceedings of the Symposium on Viscous Drag Reduction*, Dallas, Texas, 1968.

[4] Tulin, M. P., "Hydrodynamic Aspects of Macromolecular Solutions," *Proceedings of the 6th ONR Symposium on Naval Hydrodynamics*, Office of Naval Research, Washington, D. C., 1966.

[5] Lang, T. G. and Patrick, H. V. L., "Drag of Blunt Bodies in Polymer Solutions," Publ. 66-WA/FE-33, American Society of Mechanical Engineers.

[6] Rubin, H. and Elata, C., "Stability of Couette Flow of Dilute Polymer Solutions," *The Physics of Fluids*, Vol. 9, No. 10, Oct. 1966.

Experimental Data from Underwater Conical Nozzles Exhausting N_2 Gas

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THERE is particular interest in nozzle performance at great depths where the external water pressure could adversely affect nozzle performance. It has been generally assumed that the conventional ballistic equations, applicable to an air environment, can be used for nozzles exhausting under water. This assumption was first tested by Lawrence and Beauregard in 1965.[1] These authors obtained quantitative data from underwater conical nozzle experiments at 1 atm of ambient pressure. They showed that submergence altered the behavior of the exhausting gas jet. When the nozzle is operating under a flow separation condition, the separation plane oscillates back and forth along the nozzle's axis of rotation (see Fig. 1). The external jet tends to pulsate and vary in size. Jet asymmetry and initial peaks in the nozzle pressure are produced as a result of submergence.

This article presents data† obtained at simulated depths down to 700 ft. The data are presented in the form of two linear equations in order to make its use easier.

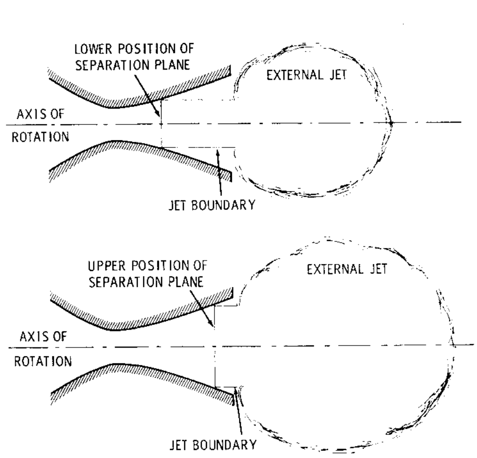

Fig. 1 Flow separation and external jet of gas exhausting from a conical nozzle submerged under water.

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† Data collected by E. K. Lawrence and author.

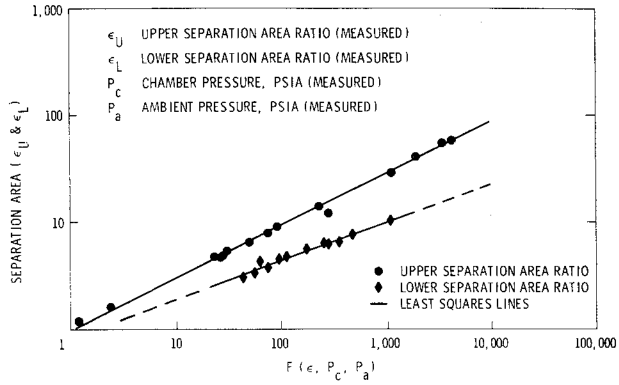

Fig. 2 Measured separation area ratio as a function of pressure for a conical nozzle exhausting nitrogen under water.

Depth simulation was carried out in the Naval Ordnance Laboratory (NOL) horizontal pressure vessel (HPV). A compressed gas facility was designed to permit operation of a compressed nitrogen gas rocket inside the 15,000 gal hydrostatic HPV.[1] Compressed nitrogen stored at 6000 psi permitted a maximum flow rate of 2.8 lb/sec using a 0.5-in. throat diameter nozzle. This corresponds to a steady-state rocket chamber pressure of 2500 psia and a chamber temperature of 50°F. A pebble bed storage heater was designed to maintain the required temperatures. The nozzle was made of Lucite so that the separation plane could be photographed. The throat diameters of the nozzles were 0.077 and 0.25 in. with a 15° half-angle. All nozzles were assembled to the same rocket chamber for firing.

Measurements were taken of the rocket chamber pressure and of the pressure in the HPV for each test. The HPV pressure remained essentially constant for the duration of each test. High-speed movies were taken of the separation plane. In almost every test, the separation plane oscillated back and forth along the nozzle axis. The lower and upper positions of the separation plane were obtained from the high-speed movie films. Thus, the data are presented in terms of the lower- and upper-separation area ratios, the chamber pressure and ambient pressure.

The data are represented by

$$\epsilon_U^{2.054} = 0.977 \left[44.02 \left(\frac{P_c^{0.588}}{P_a^{1.099}} \right) \epsilon_U - 362.4 \left(\frac{P_c^{1.170}}{P_a^{2.179}} \right) \right] \tag{1}$$

and

$$\epsilon_L^{2.788} = 0.588 \left[92.03 \left(\frac{P_c^{0.425}}{P_a^{0.833}} \right) \epsilon_L - 211.0 \left(\frac{P_c^{0.724}}{P_a^{1.338}} \right) \right] \tag{2}$$

where ϵ_U = upper-separation area ratio; ϵ_L = lower-separation area ratio; P_a = ambient pressure, psia; P_c = chamber pressure of rocket, psia. Equations (1) and (2) are equations that uniquely represent the experimental data (see Fig. 2). In Fig. 2, $F(\epsilon, P_c, P_a)$ represents Eq. (1) divided by 0.977 and Eq. (2) divided by 0.588. These equations do not necessarily state explicit relationships between the variables but provide an excellent means of interpolation and comparison of data. The following pressure (psia) limits should be observed when using these equations, $150 < P_c < 2400$ and $15 < P_a < 302$.

Reference

[1] Lawrence, E. K. and Beauregard, R. L., "A Research Program for the Study of Underwater Rocket Performance," AIAA Paper 65-485, San Francisco, Calif., 1965.